Exploratory Examination of the Functional Form of Safety Performance Functions of Urban Freeways

Jake Kononov, Ph.D., P.E. Colorado Department of Transportation Director of Research 4201 E. Arkansas Denver, Colorado 80222 Phone 303-757-9973 Fax 303-757-9974 Jake.Kononov@dot.state.co.us

Barbara Bailey, Ph.D. San Diego State University Assistant Professor Department of Mathematic and Statistics 5500 Campanile Drive San Diego, CA 92182-7720 <u>BABailey@Sciences.SDSU.edu</u>

Bryan K. Allery, P.E. Colorado Department of Transportation Safety Engineering and Analysis Group Manager 4201 E. Arkansas Denver, Colorado 80222 303-757-9967 Bryan.Allery@dot.state.co.us

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ABSTRACT

Safety Performance Functions are accident prediction models that relate traffic exposure, measured in Annual Average Daily Traffic (AADT) to safety, measured in the annual number of accidents per mile (accidents/mile per year). Review of literature on the development of Safety Performance Functions (SPF) suggests that the focus of most modeling efforts is on the statistical technique and the underlying probability distribution with only a limited consideration given to the nature of the phenomenon itself. In this study Neural Networks have been used to identify the underlying relationship between safety and exposure. The modeling process was informed by the consideration of the traffic operations parameters described by the Highway Capacity Manual (HCM). The shape of the SPF is best described by a sigmoid reflecting doseresponse-like relationship between safety and traffic demand on urban freeways. We observed that on un-congested segments the number of crashes increases only moderately with increase in traffic; however, once some critical traffic density is reached, the number of crashes begins to increase at a much faster rate with increase in traffic. This phenomenon is reflected by a steeper gradient of the SPF. Further examination of the SPF suggests that on segments with high AADT (LOS-F during peak period), the function begins to level off, reflecting decrease in accident rates related to a high degree of congestion and significant reduction in operating speeds. Relating safety to the degree of congestion suggests that safety deteriorates with the degradation in the quality of service expressed in terms of the Level of Service (LOS).

Two roads diverged in a wood And I took the one less traveled by And that has made all the difference

Robert Frost, Road less traveled

Introduction

Safety Performance Functions are accident prediction models that relate traffic exposure, measured in AADT, to safety, measured in the number of accidents over a unit of time (accidents/mile per year). Much substantive and comprehensive work in the area of accident modeling was undertaken by Hauer and Persaud (1), Hauer (2), Lord, Washington and Ivan (3) and Abdel-Aty and Radwan (4). Details concerning dataset preparation and model fitting for the development of Safety Performance Functions were described by Kononov and Allery (5).

Review of extant literature on the development of Safety Performance Functions (SPF) suggests that the focus of most modeling efforts is on the statistical technique and underlying probability distribution. Poch and Mannering (6) concluded that the Negative Binomial regression is a powerful predictive tool and should be increasingly applied in future accident frequency studies.

Shankar et al. (7) used both the Poisson and Negative Binomial distributions to evaluate the effects of roadway geometrics and environmental factors on rural accident frequency in Washington State. They used Negative Binomial assumptions when data were over-dispersed and Poisson when not.

Abdel-Aty and Radwan (4) observed that most of the accident data is over-dispersed pointing to the need for a correction to Poisson assumptions and correctly concluded that the Negative Binomial Formulation is superior to the more restrictive Poisson formulations.

Miaou (8) suggested that Poisson model assumptions should be used to establish initial relationship between highway data and accidents and if over-dispersion is found a Negative Binomial regression model can be explored.

Lord, Washington and Ivan (3) concluded that Poisson and Negative Binomial models serve as statistical approximation to the crash process. Poisson models serve well under nearly homogeneous conditions, while Negative Binomial models serve better in all other cases. They also suggested that it may be preferable to begin to develop models that consider fundamental process of a crash and avoid striving for best fit models in isolation.

There is clearly a consensus among researchers that underlying randomness is well described by the Poisson or Negative Binomial distributions. The underlying phenomenon itself, however, is not well understood.

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Harwood in (9) concluded the following: "...it would be extremely valuable to know how safety varies with V-C ratio and what V-C ratios provide minimum accident rate. Only limited research has been conducted on the variation of safety with V-C ratio. More research of this type is needed, over a greater range of V-C ratios, to establish valid relationships between safety and traffic congestion to provide a basis for maximizing the safety benefits from operational improvement projects."

Hall and Pendleton in (10) observed that: "The implications of the existence of a definite relationship between traffic accident rates and the ratio of current or projected traffic volume to capacity is quite significant. Knowledge of any such relationship would help engineers and planners assess the safety implications both of projected traffic growth on existing highways and of highway improvements designed to increase capacity."

To date the focus of the modeling efforts has been on the random variability with only a limited consideration of a systemic component. Selection of the functional form is heavily influenced by the choice of functions available in the software package used by the modeler. Accidents on an urban freeway are a byproduct of a traffic flow; therefore, observing changes in the flow parameters may give clues about probability of accident occurrence and changes in accident frequency. Hauer (2) observed that there is no reason to think that underlying phenomenon follows any simple mathematical function. Use of the Neural Networks in this study offers an opportunity to explore the underlying relationship between variables without being limited by pre-selected mathematical function. Neural Networks are not constrained by the underlying distributional assumptions and learn by example, inferring a model from training data. In this study traffic operations parameters described by the Highway Capacity Manual (HCM) (11) were used to inform the SPF development process.

DATASET PREPARATION AND MODEL DEVELOPMENT

Five years of accident data from Colorado, California and Texas were used to develop Safety Performance Functions (SPF) for the selected multilane urban freeways. California data was obtained from the Highway Safety Information System (HSIS), Colorado and Texas datasets were provided by their departments of transportation. All of the accidents that occurred on ramps and crossroads were removed prior to fitting of the models, which left only accidents occurring on the freeway mainline itself. Two kinds of SPFs were calibrated for Colorado and California; one for the total number of accidents and the other for crashes involving injury or fatality. Due to data availability only total accidents SPF models were calibrated for Texas.

SPFs were developed using Neural Networks that is a subset of a general class of nonlinear models. We used Neural Networks to analyze the data which consists of observed, univariate responses Y_i known to be dependent on a corresponding onedimensional inputs x_i . Neural Networks are not constrained by a pre-selected functional form and specific distributional assumptions. For our application, Y_i = Accidents Per

Mile Per Year (APMPY) and x_i = Annual Average Daily Traffic (AADT). The model becomes:

$$Y_i = f(x_i, \theta) + e_i$$

where

 $f(x_i, \theta)$ = the nonlinear function relating Y_i to the independent variable x_i for the *ith* observational unit,

 θ = a *p*-dimensional vector of unknown parameters, and

 e_i = is a sequence of independent random variables.

The goal of the nonlinear regression analysis is to find the function f that best reproduces the observed data. A form of the response function used in many engineering applications is a feed forward neural network model with a single layer of hidden units. The form of the model is:

$$f(x,\theta) = \beta_0 + \sum_{k=1}^{K} \beta_k \varphi(x \ \gamma_k + \mu_k)$$

Where

 $\varphi(u) = e^{u} / (1 + e^{u})$ - a logistic distribution function β_{k} = are known as connection weights and $\beta_{0}, \beta_{1}, \gamma_{1}, u_{1}$ = the parameters to be estimated μ_{k} = the biases, Ripley (10).

 μ_k – the blases, Ripley (10).

K = the number of hidden units

The function f is a very flexible nonlinear model used in this application to capture the overall shape of the observed data. The function $\varphi(u)$ is a logistic distribution function. When K = 1, there is one hidden unit. In this case, the function performs a linear transformation of the input x and then applies the logistic function $\varphi(u)$, followed by another linear transformation. The overall result is a very flexible nonlinear model.

The parameter vectors $\beta_0, \beta_1, \gamma_1, u_1$ for each dataset are unknown and will be estimated by nonlinear least squares. The complexity for this application is the number of hidden units *K* in the model. We have chosen *K* = 1 based on general understanding of the underlying physical phenomenon. Additionally the complexity of the model is most often chosen based on the generalized cross validation (GCV) model-selection criterion. Cross-validation is a standard approach for selecting smoothing parameters in nonparametric regression described by Wahba (11). **Figures 1** through **4** represent total

crash SPFs and model fit information for the selected multilane freeways in Colorado, California and Texas. The R² parameter, predicted values from the model versus the residuals and the root mean squared error (RMSE) are also given. The residuals exhibit a pattern of increased variance as the AADT values increase. This is to be expected given the overall pattern of the data. Overall model fit to the data is quite reasonable.







Figure 2 California 6-Lane Freeway SPF and Model Fit Information



Figure 3 California 8-Lane Freeway SPF and Model Fit Information





Figure 4 Texas 8-Lane Freeway SPF and Model Fit Information

SPFs for multilane freeways in different states are different due to different reporting thresholds, climate and other local factors, yet sigmoid functional shapes of the Safety Performance Functions generated by the Neural Network regression are similar. The shape reflects a relationship similar to a dose-response curve found in medicine and pharmacology, as well as other sciences. In all cases, accident data for urban freeways exhibited extra-variation or over-dispersion relative to the Poisson model.

RELATING CHANGES IN ACCIDENT RATES WITH CHANGES IN THE SHAPE OF THE SAFETY PERFORMANCE FUNCTION – A BRIEF OVERVIEW

Accident rates change with AADT, and SPF reflects how these changes take place. Higher rates within the same SPF mean less safety than lower rates. Any accident frequency derived from the SPF expressed in accidents per mile per year (APMPY) can

be easily converted into accident rates measured in accidents per million vehicle miles traveled (acc. per mvmt). For instance, the Colorado SPF calibrated for 6-lane urban freeways (Figure 5) 120,000 AADT is expected to produce on the average 56 accidents per mile per year, 56 acc/mi annually can be directly converted to the accident rates as follows:

(56 acc / mile / year) x 1,000,000





It is of interest to observe how changes in the accident rates are reflected by the shape of the safety performance function. Let's presume that (A) and (B) are values from the Fi, a Safety Performance Function representing a multilane freeway. Accident rate (R) at AADT_A and AADT_B can be expressed as follows:

Accident Rate at $A = R_A$

$$R_A = \frac{A}{AADT_A} * C$$

Accident Rate at B= R_B

$$R_{B} = \frac{B}{AADT_{B}} * C$$

 $C = \frac{1,000,000}{1mile * 365 days / year}$

As we make a transition from A to B the number of accidents is increasing with AADT; however, the accident rate itself can remain the same, decrease or increase depending on the shape of the SPF. **Figure 6** graphically represents each scenario.





Rate @
$$A = Rate @ B$$

If A and $B \in F_i$ and $\frac{A}{AADT_A} = \frac{B}{AADT_B} \Rightarrow R_A = R_B$

In this case as a transition is made from point A to point B of the SPF the accident rate at A is the same as the accident rate at B. The number of accidents increases with AADT in such a way that the ratio of crashes to exposure at points A and B is preserved. It is reflected by a relatively moderate gradient in the shape of the function.

Rate Increases

If A and
$$B \in F_i$$
 and $\frac{A}{AADT_A} < \frac{B}{AADT_B} \Rightarrow R_A < R_B$

In this case as a transition is made from point A to point B of the SPF the accident rate is increasing. The number of accidents increases with AADT in such a way that the ratio of crashes to exposure is increased. It is reflected by a relatively steep gradient in the shape of the function.

Rate Decreases

If A and
$$B \in F_i$$
 and $\frac{A}{AADT_A} > \frac{B}{AADT_B} \implies R_A > R_B$

In this case as a transition is made from point A to point B of the SPF the accident rate is decreasing. The number of accidents is increasing with AADT, but AADT is increasing faster. It is reflected by a very mild upward gradient of the SPF.

Figure 7 shows changes in accidents rates observed on 6-lane urban Colorado freeways from the low to high range of AADT. For the SPF representing total crashes the accident rate is more than doubled as a transition is made from 60,000 to 150,000 AADT.



Figure 7 Changes in Accident Rates within SPF (Total Accidents and Injury and Fatal Crashes Only)

For the injuries and fatal crashes SPF the accident rate increases from 0.23 Acc/MVMT to 0.37 Acc/MVMT (65%) as AADT increases from 60,000 to 150,000. The sigmoid functional shape of the SPF has 2 critical points where rate of change in the gradient of the function is significantly altered. These points were located using *a sliding interval analysis in the framework of the numerical differentiation technique* described by Rao (13). **Figure 8** provides a generalized diagram of the process.



Figure 8 Numerical Differentiation / Sliding Interval Diagram

A scanning interval of 2 Δx is incrementally moved forward along the function. In each position of the interval a numerical derivative is computed on both sides of the sliding point. The ratio R of estimated derivatives on both sides of the sliding point in the middle of an interval is calculated at every position of the scanning interval until it reaches a predetermined critical value of R_c. For most SPFs R_c \geq 1.5. Selection of a scanning interval Δx is data driven and was found to be effective at 20,000 AADT for multilane urban freeways.

When a transition is made from a milder into a steeper reach of the Safety Performance Function the ratio R was computed as follows:

$$f_1 \& f_2 \in F, \Delta_1 = \Delta_2 = \Delta_x$$
$$\frac{\Delta y_2 / \Delta x}{\Delta y_1 / \Delta x} \approx \frac{\sum df_2}{\sum df_1} = R \ge R_c$$

When a transition is made from a steeper to milder reach of the Safety Performance Function the ratio R was computed as follows:

$$f_3 \& f_4 \in F, \Delta_1 = \Delta_2 = \Delta_x$$
$$\frac{\Delta y_3 / \Delta x}{\Delta y_4 / \Delta x} \approx \frac{\sum df_4}{\sum df_4} = R \ge R_c$$

$$\begin{array}{c} 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 21 \\ 22 \\ 23 \\ 24 \\ 25 \\ 26 \\ 27 \\ 28 \\ 29 \\ 30 \\ 31 \\ 32 \\ 33 \\ 34 \\ 35 \\ 36 \\ 37 \\ 38 \\ 39 \\ 40 \\ 41 \\ 42 \\ 43 \\ 44 \\ 45 \\ 46 \\ 47 \\ 48 \end{array}$$

Using sliding interval and numerical differentiation technique, critical points were identified at AADT of 90,000 and 150,000 for SPF (Total) and 90,000 and 140,000 for SPF (Inj.+Fatal).

Relating Changes in Freeway Flow Parameters with Changes in Accident Rates Reflected by the Shape of the SPF

In an effort to relate freeway flow parameters such as speed (v) and density (d) during peak period associated with the changes in the shape of the SPF, HCM 2000 (12) methodology was used. The following assumptions typical of the urban freeway environment were used:

DHV (Design Hourly Volume) =10 % of AADT for AADT <130,000 DHV(Design Hourly Volume)=8% of AADT for AADT>130,000 PHF (Peak Hour Factor) = 0.9 %Truck during peak period = 2% Terrain – Level Lane Width = 12 ft Shoulder Width > 6 ft. Interchange spacing =1 interchange / mile

The results of the HCM analysis were superimposed onto SPF and are presented in **Figure 9**. Traffic density at 90,000 AADT identified previously as a critical point on the SPF can be viewed as a *Critical Density*, beyond which accidents increase at a faster rate. A portion of the SPF to the left of Critical Density can be viewed as a Sub-Critical Zone where accidents increase gradually with AADT. Traffic density at 150,000 AADT can be viewed as a Super-Critical Density beyond which accidents increase very gradually with AADT and accident rates level off or even decline. A portion of SPF to the right of Super-Critical Density can be viewed as a Super-Critical Zone. A portion of the SPF between Critical and Super-Critical Densities can be termed Transitional Zone.

It is of interest to note that as AADT increases from 60,000 to 90,000, traffic density increases by 50% (from 16 pc/mi/ln to 24 pc/mi/ln.), while operating speeds remain almost the same (70 and 69 mph). It is not unreasonable to assume that if operating speeds remain high and traffic density is increased by 50%, accident probability is also increased. The freeway environment *becomes much less forgiving of driving errors and road rage-like behavior with increase in density of traffic at freeway speeds.* The SPF reflects that past the AADT of 90,000 the number of crashes increases at a much faster rate with increase of AADT.



Figure 9 6-Lane Freeway SPF – Changes in Traffic Speed on Density

A possible explanation is that traffic has *reached some Critical Density* beyond which notably higher accident rates are observed. This increase in the rates is manifested by the steeper gradient of the SPF.

Examination of the SPF in concert with traffic operations parameters suggests that when freeways are not congested and traffic density is low the number of crashes increases only moderately with increase in traffic. That is why initially the slope of the SPF is relatively flat; however, once Critical Density is reached, the number of crashes begins to increase at a much faster rate with increase in traffic. *Attainment of the Critical Density can be viewed as a critical mass-like phenomenon in physics. Mix of density and speed of traffic is such that probability of a crash is substantially increased, thus a steep reach of the SPF.*

Further examination of SPF suggests that that past the point of Super-Critical Density (*AADT of 150,000*) the function begins to level off, reflecting only moderate increase in accidents and decrease in accident rates related to high degree of congestion and significant reduction in the operating speeds. Density exceeds 45 vehicles per mile per lane and speeds are below 52 mph which corresponds to a LOS-F.

Figures 10 and 11 show the boundaries of the Levels of Service (LOS) during the peak period superimposed onto the SPFs for the total and injury and fatal crashes. The LOS boundaries during peak periods were estimated using HCM under the same default assumptions as earlier. Average accident rates for the total and injury and fatal crashes were computed for each Level of Service (LOS) and are also provided in Figures 10 and 11.







Figure 11 SPF 6-Lane Injuries and Fatal Changes in Accident Rates with Level of Service

Integrating LOS and accident rates in the SPF framework allows us to quantitatively relate safety to the degree of congestion. The data shows that total as well as injury

and fatal crash rates increase with AADT and that it is significantly safer to travel on urban freeways that operate at the LOS-C or better during the peak period than on more congested facilities. This knowledge has important implications on the philosophy and policy of transportation planning and highway design criteria.

Summary

Use of the Neural Networks lends itself well to studying the systemic component of the relationship between safety and traffic exposure on multilane urban freeways. The functional shape of the Safety Performance Function is well described by a sigmoid curve reflecting a dose-response like relationship found in medicine and pharmacology, as well as other sciences. In all cases, accident data for urban freeways exhibited extra-variation or over-dispersion relative to the Poisson model.

We observed that on un-congested segments the number of crashes increases only moderately with increase in traffic; however, once some critical traffic density is reached, the number of crashes begins to increase at a much faster rate with increase in traffic. This phenomenon is reflected by a steeper gradient of the SPF. High density of traffic in the high range of AADT is associated with approaching Super-Critical Density and leveling off of the SPF, reflecting a high degree of congestion and reduction in operating speeds.

Relating different Levels of Service (LOS) during peak periods with accident rates within SPF shows that total as well as injury and fatal crash rates increase with congestion. *This observation suggests that peak spreading and congestion pricing have potential for safety improvement in addition to more obvious mobility benefits.* Understanding of the relationship between the LOS and accident rate can be used to inform public policy, transportation planning process as well as highway design criteria. It offers an important insight into the relationship between safety and mobility that will improve quality of decisions made by the practicing engineers, planners and elected officials.

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