Relationship Between Traffic Density, Speed, and Safety and Its Implications for Setting Variable Speed Limits on Freeways

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Speed-flow relationships for a typical basic freeway segment are understood well at present and are documented by the successive editions of the Highway Capacity Manual. Recent freeway studies showed that speed on freeways was not affected by low to midrange traffic flow. Increases in flow and density without a reduction in speed have a significant influence on safety. However, current literature lacks constructive discussion of this influence. Empirical examination of the relationship between flowdensity, speed, and crash rate on selected freeways in Colorado suggests that as flow-density increases, the crash rate initially remains constant until a certain critical threshold combination of speed and density is reached. Once this threshold is exceeded, the crash rate rises rapidly. The rise in crash rate may be caused by flow compression without a notable reduction in speed: resultant headways are so small that drivers find it difficult or impossible to compensate for error and avoid a crash. This paper calibrates performance functions for corridor-specific safety that relate crash rate to hourly volume-density and speed and proposes an algorithm for a variable speed limit intended to slow traffic in real time in advance of a high speed-high density operational regime. Deployment of such an algorithm has the potential to improve safety and reduce travel time variability.

We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances.

-Isaac Newton

Speed–flow and density–flow relationships for a typical basic freeway segment are well understood at present and are documented by the successive editions of the *Highway Capacity Manual* (1). Recent freeway studies show that speed on freeways is not affected by flow in the low to midrange. Increases in flow and density without a notable reduction in speed have a significant influence on safety. This influence, however, has not been studied extensively and has attracted only limited interest from researchers to date. Lord et al. observed that most of the research has focused on determination of the relationship between crashes and annual average daily traffic, whereas little attention has been focused on the relationships of vehicle density, level of service (LOS), vehicle occupancy, volume-to-capacity ratio, and speed distribution (2). Zhou and Sisiopiku found that crash rates typically follow a U-shaped relationship when they are plotted as a function of the volume-to-capacity ratio (3).

Traditional safety performance functions (SPFs) relate accident occurrence to average annual daily traffic. Persaud and Dzbik observed that a difficulty with this approach is that a freeway with intense flow during peak periods would have a different accident potential than a freeway with the same average annual daily traffic but with flow evenly spread out throughout the day (4). Kononov et al. observed that on uncongested freeways the number of crashes increases moderately with an increase in traffic; however, once some critical traffic density is reached, the number of crashes begins to increase at a much higher rate with an increase in traffic (5). Garber and Subramanyan related crashes to lane occupancy and concluded that peak crash rates do not occur during peak flows (6).

Harwood noted that it would be extremely valuable to know how safety varies with the volume-to-capacity ratio and what volumeto-capacity ratios provide the minimum accident rate (7). Hall and Pendleton observed that knowledge of the definite relationship between the volume-to-capacity ratio and crash rate would help engineers and planners assess the safety implications of highway improvements designed to increase capacity (8). Lord et al. concluded that "despite overall progress, there is still no clear understanding about the effects of different traffic flow characteristics on safety" (2).

Figure 1, which is Exhibit 23-3 from the 2000 edition of the Highway Capacity Manual, shows the speed-volume-density relationship and LOS for basic freeway segments (1). It shows that drivers on modern freeways slow down very little or not at all as the LOS deteriorates from A to D. When one considers that the perception-reaction time and vehicle characteristics remain unchanged, even though considerably more vehicles are in the same space traveling at substantially the same speed as before, an increased probability of crash occurrence is highly plausible. This increase would be reflected by changes in the crash rate. For instance, a freeway with a free-flow speed of 70 mph at Point 1 carrying 600 passenger cars per hour per lane [Volume 1 (V_1)] has density d_1 of 8.6 passenger cars per mile per lane (pc/mi/ lane) and operates at LOS A. When congestion builds up to 1,750 pc/ mi/lane (V_2) (the boundary between LOS C and LOS D), the resulting density (d_2) rises to 26 pc/mi/lane and the operating speed drops only slightly to 68 mph.

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FIGURE 1 Speed-flow curves and LOS for basic freeway segments (pc/h/ln = passenger cars per hour per lane; S = speed; V = volume)(1).

As a transition from Point 1 to Point 2 is made, densities that are almost three times greater and a decrease in speed of only 3% are observed. When these flow parameters are examined for a freeway with a free-flow speed of 55 mph, the volume rises from 600 vehicles per hour (density = 10.9 pc/mi/lane) to 1,750 vehicles per hour (density = 31.8 pc/mi/lane) without any speed reduction. Compression of flow without a corresponding reduction in speed is likely to have an adverse effect on safety; calibration of this effect is the focus of this paper. Furthermore, use of variable speed limits (VSLs) to mitigate this problem is explored.

MODEL DEVELOPMENT

Data Set Preparation

Hourly volume, operating speed, and free-flow speed data were collected from existing automatic traffic recording stations on four-lane freeways and a segment of Interstate 70 that carries traffic to ski resorts in mountainous terrain around the Denver, Colorado, metropolitan area. The main-line crash history was obtained from the Colorado Department of Transportation crash database for every hour over a 5-year period (2001 to 2006) for every freeway in the data set. All crashes that occurred on ramps and crossroads were removed before fitting of the models.

Matching of the hourly volume on every segment with its crash history enabled computation of the crash rate for every hour of the 24-h period for all freeways in the data set. A graph demonstrating changes in volume and crash rates throughout the day on typical four-lane freeways in the Denver area is presented in Figure 2.

Nearly 60% of all crashes that occurred between midnight and 5 a.m. but only 4% of those that occurred during the rest of the day involved alcohol use, drug use, or falling asleep at the wheel. Such a dramatic difference in driver performance abilities and crash causality suggests that they are qualitatively different phenomena. A mix of impaired and fatigued drivers with low volumes produces very high crash rates between midnight and 5 a.m. compared with the daytime safety performance of the same segments. This finding

may possibly explain the U-shaped relationship identified by Zhou and Sisiopiku (3).

The impaired driver issue, a largely behavioral problem, is distinct from issues that present problems near or at peak times. With recognition of this, a portion of the data set containing safety performance data for the period between midnight and 5 a.m. was removed before calibration of the corridor-specific SPFs. Figure 2 also suggests that the afternoon peak is characterized by higher crash rates than the morning peak. It may be speculated that commuters are more fatigued, less focused on the driving task, and eager to get home from work. The higher crash rate may also be attributed to the larger number of secondary crashes that result from the longer duration of the p.m. peak period. With this in mind, separate corridor-specific SPFs for morning and afternoon peak periods on urban freeways and seasonal SPFs for the section of I-70 carrying traffic to ski resorts were calibrated.

Relating Basic Kinematics to Flow Theory

A possible way to explore the relationship between safety and traffic flow parameters is to examine the average distance between vehicles available at different combinations of density and speed and to compare it with the distance required to slow down to avoid a crash because of a sudden change in traffic flow conditions or driver error. The average distance between vehicles can be approximately expressed as a function of density:

$$h_i = c \frac{1}{d_i} \tag{1}$$

where

- h_i = average distance between cars under operational conditions *i*,
- c =constant that approximately accounts for the distance occupied by vehicles, and
- d_i = density (number of passenger cars per mile per lane) under operational conditions *i*.

The basic motion equation for deceleration is



FIGURE 2 Changes in volume and crash rate over the 24-h period on Denver area urban freeways (veh/h/ln = vehicles per hour per lane; inj & fat = injuries and fatalities).

$$D_r = \frac{S_i^2 - S_e^2}{2a}$$
(2)

where

- D_r = distance required to decelerate from S_i to S_e ,
- S_i = initial speed,
- S_e = end speed, and
- a = rate of deceleration (assumed constant).

Under safe operational conditions, the distance required to slow down to avoid a crash must be less than the average available distance between vehicles; therefore,

$$\frac{S_i^2 - S_e^2}{2a} < h_i = c \frac{1}{d_i}$$
(3)

By application of Equation 3 to the back-of-the-queue scenario frequently encountered on freeways, where S_e is equal to 0, Equation 3 becomes

$$\frac{S_i^2}{2a} < c\frac{1}{d_i} \tag{4}$$

This can now be modified as follows:

$$d_i S_i^2 < c2a \tag{5}$$

The right side of the equation can be viewed as a constant, C_0 , and therefore, the equation becomes the threshold inequality presented below:

$$d_i S^2 < C_0 \tag{6}$$

Another possible scenario may involve a sudden need to decelerate because of a slower-moving vehicle ahead. The time t_{i-e} required to decelerate (at an assumed constant rate) from S_i to S_e satisfies the following basic kinematics equation:

$$t_{i-e} = \frac{S_i - S_e}{a} \tag{7}$$

During time t_{i-e} , a slower-moving vehicle traveling at speed S_e will travel the distance D_e , which can be expressed as

$$D_e = S_e t_{i-e} = \frac{S_e(S_i - S_e)}{a}$$
(8)

During the process of deceleration, S_e can be expressed as some proportion p of S_i :

$$S_e = pS_i \tag{9}$$

Substitution of S_e from Equation 9 into Equation 8 gives the following expression:

$$D_e = \frac{pS_i(S_i - pS_i)}{a} = \frac{pS_i^2 - p^2S_i^2}{a} = \frac{pS_i^2(1-p)}{a}$$
(10)

As the faster-moving vehicle decelerates from S_i to S_e , it will travel distance D_r described by Equation 2.

With the replacement of S_e with pS_i , distance D_r can be expressed as follows:

$$D_r = \frac{S_i^2 - p^2 S_i^2}{2a} = \frac{S_i^2 (1 - p^2)}{2a} = \frac{S_i^2 (1 - p)(1 + p)}{2a}$$
(11)

A relative change in distance (Δ) between two vehicles over the time of deceleration from S_i to S_e is computed below:

$$\Delta = \frac{S_i^2 (1-p)(1+p)}{2a} - \frac{2pS_i^2 (1-p)}{2a}$$
$$= \frac{S_i^2 (1-p)(1+p-2p)}{2(a)} = \frac{S_i^2 (1-p)^2}{2a}$$
(12)

The requirement that Δ be less than some multiple c_1 of the average distance between vehicles (h_i) produces the threshold inequality presented below:

$$\frac{S_i^2 \left(1-p\right)^2}{2a} < c_1 h_i = c_2 \frac{1}{d_1}$$
(13)

or

$$d_i S_i^2 < C \tag{14}$$

where c_1, c_2, p, a , and C are constants with respect to speed and volume.

Comparison of the available distance between cars traveling at speed S_i with the requisite distance to avoid a crash via dS^2 does not address all modes of crash occurrence. This model represents only a simplified version of reality. However, when one considers that more than 70% of freeway crashes are rear-end crashes and sideswipes, the model addresses the most prevalent mechanisms of crash occurrence.

The appearance of density and speed terms in the inequality presented above motivates the consideration of density in concert with speed as the relationship between flow characteristics and safety is explored by the use of neural networks. In particular, appearance of density and speed terms suggests that properties beyond volume Vequal to dS should be considered. Use of V alone runs counter to the expectation that a segment with a high volume produced by a high density at low speed may have a different crash rate than the same segment with the same volume produced by, say, half the density and twice the speed. The discussion that follows uses the form of the threshold inequality derived above. This form should be verified or modified on the basis of additional empirical evidence.

Neural Networks

Corridor-specific SPFs relating freeway flow parameters to the crash rate were developed by the use of neural networks, which are a subset of a general class of nonlinear models. Neural networks were used to analyze the data, which consisted of observed, univariate responses (Y_i) known to be dependent on corresponding onedimensional inputs (x_i). Neural networks are not constrained by a preselected functional form and specific distributional assumptions. For the current application, Y_i is the crash rate (measured in numbers of crashes per million vehicle miles traveled) and x_i is dS^2 , where d is density (measured in number of passenger cars per mile per lane) and S is speed (measured in miles per hour). The model becomes

 $Y_i = f(x_i, \theta) + e_i$

where

- $f(x_i, \theta)$ = nonlinear function relating Y_i to the independent variable x_i for the *i*th observational unit,
 - $\theta = p$ -dimensional vector of unknown parameters, and
 - e_i = sequence of independent random variables.

The goal of the nonlinear regression analysis is to find the function f that best reproduces the observed data. A form of the response function used in many engineering applications is a feed-forward neural network model with a single layer of hidden units. The form of the model is

$$f(x, \theta) = \beta_0 + \sum_{k=1}^{K} \beta_k \varphi(x \gamma_k + \mu_k)$$

where

$$\beta_0, \beta_1, \gamma_i, \text{ and } \mu_1 = \text{ parameters to be estimated for } i = 1, \dots, K;$$

 $\beta_k = \text{ connection weights;}$

 $\varphi(u) = e^{u}/(1 + e^{u}) =$ logistic distribution function;

- μ_k = biases (9);
- k = subscript for parameters to be estimated; and

K = number of hidden units.

The function *f* is a flexible nonlinear model used in this application to capture the overall shape of the observed data. The function $\varphi(u)$ is a logistic distribution function. When *K* is equal to 1, one unit is hidden. In this case, the function performs a linear transformation of input *x* and then applies the logistic function $\varphi(u)$, followed by another linear transformation. The result is still a very flexible nonlinear model.

The parameters β_0 , β_1 , γ_i , and μ_1 for each data set are unknown and will be estimated by nonlinear least squares. The complexity for this application is the number of hidden units *K* in the model. A *K* value of 1 was chosen on the basis of a general understanding of the underlying physical phenomenon. In addition, the complexity of the model is most often chosen on the basis of the generalized crossvalidation model selection criterion. Cross validation is a standard approach for selecting smoothing parameters in nonparametric regression described by Wahba (*10*). The overall fit of the model to the data is quite good (Figures 3 to 7).

The product of traffic density (d_i) times its speed squared (S_i^2) as an explanatory variable that enables consideration of density in concert with speed as the relationship between flow characteristics and safety is examined. The data in Figures 3 to 7 reflect these relationships for several freeways in the Denver metropolitan area and a heavily traveled rural freeway in a mountainous environment.

The inventory of freeways used for the study described in this paper did not include any freeways that had volumes that exceeded 1,800 vehicles per hour per lane. This may explain why the reduction in crash rates associated with heavy congestion described by Kononov et al. is not reflected in the functional form of corridor-specific SPFs in this study (5). Furthermore, the limited range of speeds represented prevents detailed analysis of the way in which speed enters the threshold inequality. The findings presented in Figures 3 to 7 suggest that the crash rate remains relatively stable until a certain threshold value of dS^2 is reached. Once that threshold value dS^2 can thus be viewed as a corridor-specific flow crash potential indicator (FCPI), which reflects the probability of a crash for different operational regimes; that is, FCPI is equal to dS^2 .

The relationship between dS^2 (FCPI) and crash rates seems to resemble a phase change phenomenon in chemistry or critical mass in physics. A possible explanation may be that if FCPI exceeds a certain critical threshold value (FCPI_{cr}), the available headway becomes too small for the prevailing speed to allow drivers to react effectively to changing traffic conditions. Furthermore, two distinct operational regimes can be observed in Figure 8, as well as all other



FIGURE 3 Corridor-specific SPF for C-470 (p.m., four lanes, 7 mi) (crash/MVMT = numbers of crashes per million vehicle miles traveled; DS^2 = density times speed squared).



FIGURE 4 Corridor-specific SPF for C-470 (a.m., four lanes, 7 mi).



FIGURE 5 Corridor-specific SPF for I-270 (a.m., four lanes, 5 mi).



FIGURE 6 Corridor-specific SPF for I-225 (a.m., four lanes, 6 mi).



FIGURE 7 Corridor-specific SPF for I-70 eastbound, winter season (p.m., four lanes, 5 mi).



FIGURE 8 Corridor-specific SPF with Regimes 1 and 2.

corridor-specific SPFs: Regime 1, in which FCPI is less than $FCPI_{cr}$, and Regime 2, in which FCPI is greater than $FCPI_{cr}$.

Regime 1 is characterized by low to moderate densities and high speeds, in which drivers are still able to compensate for increasing density. An increased focus on the driving task may possibly explain the fact that during Regime 1 the crash rate remains stable, despite the increase in density. Regime 2 is characterized by moderate to high densities without a notable speed reduction, in which the combination of speed and density is such that more drivers are not able to compensate for driver error and avoid a crash. In Regime 2, a greater portion of near misses becomes crashes, reflected by a sharp rise in the crash rate.

A possible strategy to counteract the deficit of available deceleration distance associated with a mix of high speeds and short headways is to slow traffic down in real time by the use of VSLs.

ALGORITHM FOR VSLs

VSL control is an active traffic management strategy intended to maximize throughput, improve safety, and reduce travel time variability. According to Chang et al., a VSL system typically consists of a set of traffic sensors to collect flow and speed data, several properly located variable message signs for message display, a reliable control algorithm to compute the optimal speed for all control locations, and a real-time database as well as a communications system to convey information between all principal modules (11).

The core of the logic of VSLs is to adjust a set of speed limits dynamically to harmonize the speed transition between the upstream free-flow state and the downstream congested traffic state (11). This harmonization or smoothing of traffic flow is thought to prevent the formation of an excessive queue because of a shock-wave effect. Hegyi et al. demonstrated that the use of VSLs can be an effective strategy to increase throughput on recurrently congested freeways in Europe by reducing or eliminating the shock wave (12). The principal aim of the extensive deployment of VSLs in Europe was to improve safety and traffic operations on freeways.

In contrast to the expertise shown in Europe, the state of reliable knowledge on the safety and mobility benefits of VSLs in the United States is emerging but is limited at present. Golub et al. identified flow patterns associated with crash types by using loop detectors in California and developed a software tool for predicting crash types most likely to occur (13). Substantive and innovative work in the general area of active traffic management and VSLs in particular was done by Abdel-Aty et al. (14). With a logistic regression model, Abdel-Aty et al. have shown that observation of a high variability in speed 5 to 10 min before the crash in which the variability is represented by its coefficient of variation (standard deviation/mean) was the most significant predictor of a crash (14). By the time that speed variability is observed, however, it may be more difficult to influence the flow effectively by slowing it down. Although speed variability is strongly correlated with crashes, it may be more effective to intervene by the use of VSLs in advance of the observation of turbulence reflected by a speed differential.

The findings presented in Figures 3 to 7 suggest that when dS^2 exceeds a certain corridor-specific threshold or critical FCPI, one begins to observe a rapid deterioration of safety, demonstrated by a rise in the crash rate. The critical value of FCPI can be estimated by use of a sliding interval analysis in the framework of the numerical differentiation technique described by Rao (15). A strategy that may possibly counteract the deficit of available deceleration distance associated with a mix of high speeds and short headways is to slow traffic down in real time by the use of a VSL. This idea lends itself to the conceptual algorithm shown in Figure 9, where

- $\text{FCPI}_{o} = \text{observed flow crash potential index } (\text{FFCPI}_{o} = d_{o}S_{o}^{2}),$
 - d_o = observed density of flow (pc/mi/lane),
 - S_o = observed speed,
- FCPI_{cr} = critical corridor-specific value of freeway flow crash potential index estimated by use of corridor-specific SPF, and
 - S_r = recommended speed ($S_r = \sqrt{\text{FCPI}_{cr}/d_o}$) rounded to the nearest 5 mph.



FIGURE 9 Algorithm for VSLs.

The ideal would be to operate freeways in Regime 1 at less than critical values of FCPI; however, a final resulting operating speed will be influenced by the degree of compliance. This conceptual algorithm is intended to compute the recommended baseline speed on individual segments for which the SPF has been calibrated. In practice, the final VSL display will be informed by the real-time traffic operations upstream and downstream. Figure 10 shows how the algorithm is intended to work by combining the corridor-specific

TABLE 1 Observed Speeds and Recommended or Posted Speeds

Speed Observed (S)	Density Observed (d_o)	FCPI $(d_o S^2)$	$S_r = \sqrt{\frac{80,000}{d_o}}$	S_r^{a}
70.5	4.2	20,875	FCPI < FCPI _{cr} display posted speed limit	70
70.5	18.4	91,453	65.9	65
70.4	20.1	99,619	63.1	65
70.2	21.8	107,431	60.6	60
69.6	23.7	114,807	58.1	60
68.6	25.8	121,414	55.7	55
67.1	28.1	126,518	53.4	55
65.1	30.8	130,531	51.0	50
62.4	34.0	132,388	48.5	50
59.1	37.9	132,377	45.9	45
55.0	42.8	129,470	43.2	45

"Rounded to the nearest 5 mph and displayed on a variable message sign.

SPF with observed and recommended traffic flow parameters for a freeway with an FCPI of 80,000 and a static speed limit of 70 mph. Table 1 shows all related calculations and observed as well as recommended speeds, based on the hypothesized form of the threshold inequality.

Inclement weather adversely influences safety as well traffic operations. Although speed–flow curves for snowy and rainy conditions are provided in the *Highway Capacity Manual* (1), the impact of adverse weather on freeway safety has not been fully calibrated. Preliminary results from the I-70 corridor used in this study suggest that crash rates computed for hourly volumes during ski season are notably



FIGURE 10 Corridor-specific SPF (FCPI = 80,000) with observed and posted speeds.



FIGURE 11 VSL decision tree.

higher than crash rates for the same volumes in the summertime. When the weather is a factor, it is important to calibrate seasonally adjusted, corridor-specific SPFs to identify $FCPI_{cr}$. Figure 11 shows a decision tree reflecting the process of establishing VSLs on the basis of time of day and weather conditions.

SUMMARY

All recent freeway studies show that speed on freeways is insensitive to flow in the low to midrange. Increases in flow and density without a notable reduction in speed have a significant influence on safety. This influence, however, has not been studied extensively and has attracted only limited interest from researchers to date. Empirical examination of the safety performance of Colorado freeways as a function of dS^2 suggests that the crash rate remains relatively stable until a certain threshold is reached. The relationship between dS^2 or FCPI and crash rates seems to resemble a critical mass-like phenomenon in physics. A possible explanation may be that if FCPI exceeds a certain critical threshold value, the available headway becomes too small to allow drivers traveling the prevailing speed to react effectively to changing traffic conditions.

When basic kinematics are related to flow theory, this interpretation is shown to be consistent with a threshold based on the value of dS^2 . Further empirical investigation over a wider range of speeds will be necessary to refine the relationship between speed, density, and the threshold.

Two distinct operational regimes can be observed in all corridorspecific SPFs: Regime 1, in which FCPI is less than FCPI_{cr}, and Regime 2, in which FCPI is greater than FCPI_{cr}. Regime 1 is characterized by low to moderate density and high speeds, in which drivers are becoming more focused on the driving task and are still able to compensate for increasing density. This increased focus on the driving task may possibly explain why the crash rate remains stable in Regime 1, despite the increase in density. Regime 2 is characterized by moderate to high densities without a notable speed reduction in which the combination of speed and densities is such that many more near misses become crashes, and thus, a sharp rise in the crash rate occurs.

A possible strategy to counteract the deficit of the available deceleration distance produced by a mix of high speeds and short headways is to slow traffic down in real time by the use of VSLs. A conceptual algorithm for VSLs proposed in this paper is intended to establish recommended baseline speeds on individual freeway segments for which SPFs have been calibrated. The final VSL display will be informed by real-time traffic operation considerations. Deployment of such an algorithm has the potential to improve safety and reduce travel time variability. In addition, the underlying relationships between safety, speed, and the density of freeway flow have the potential to be integrated with various traffic simulation software packages currently in use.

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